Radar Systems Engineering
Lecture 11
Waveforms and Pulse Compression

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Guest Lecturer

Block Diagram of Radar System

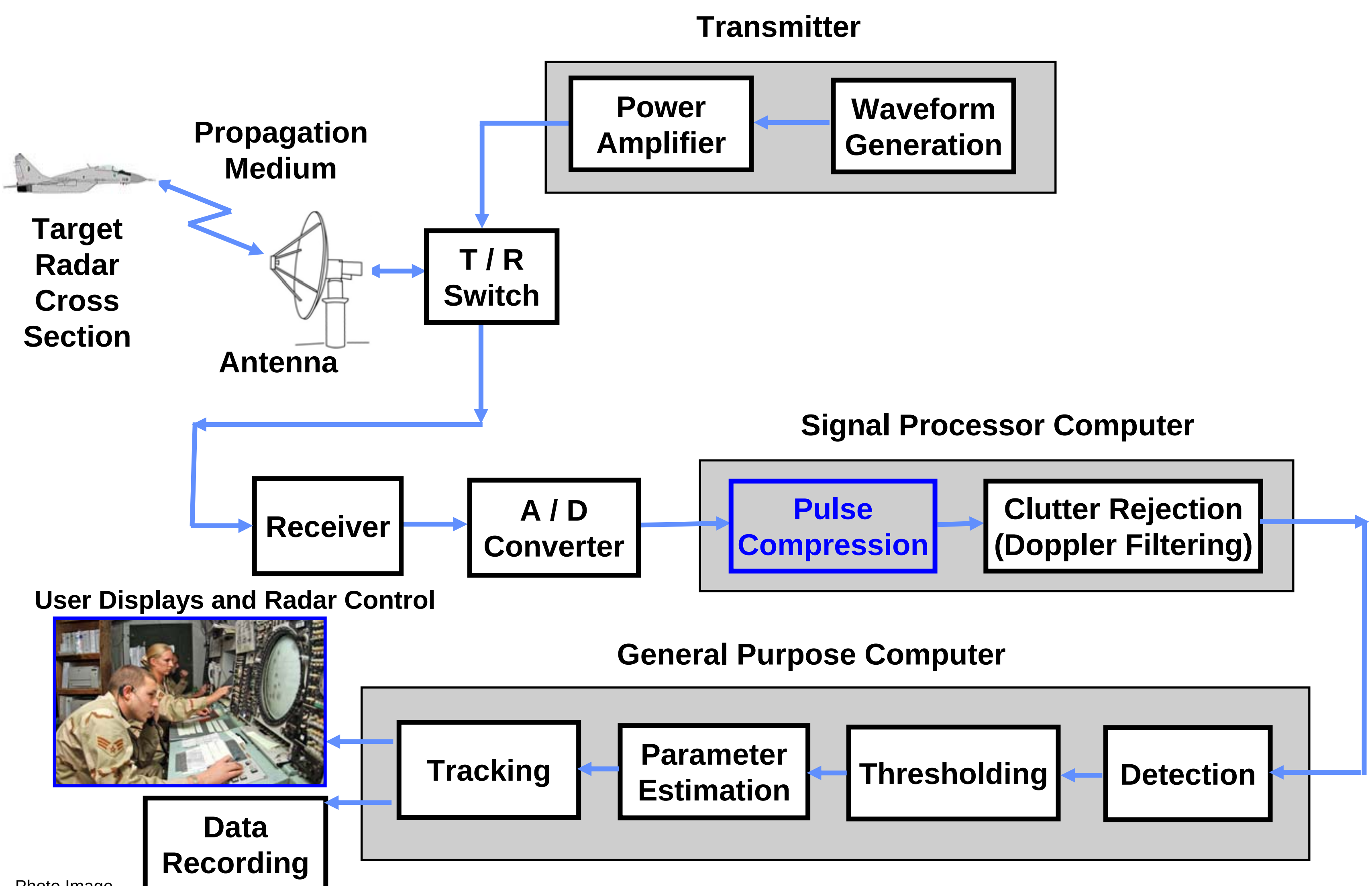
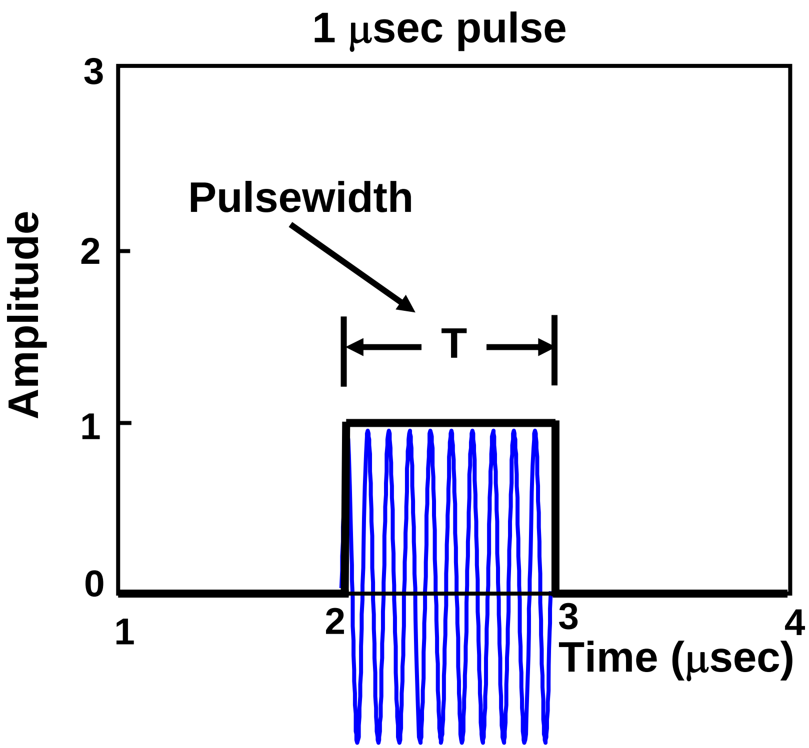


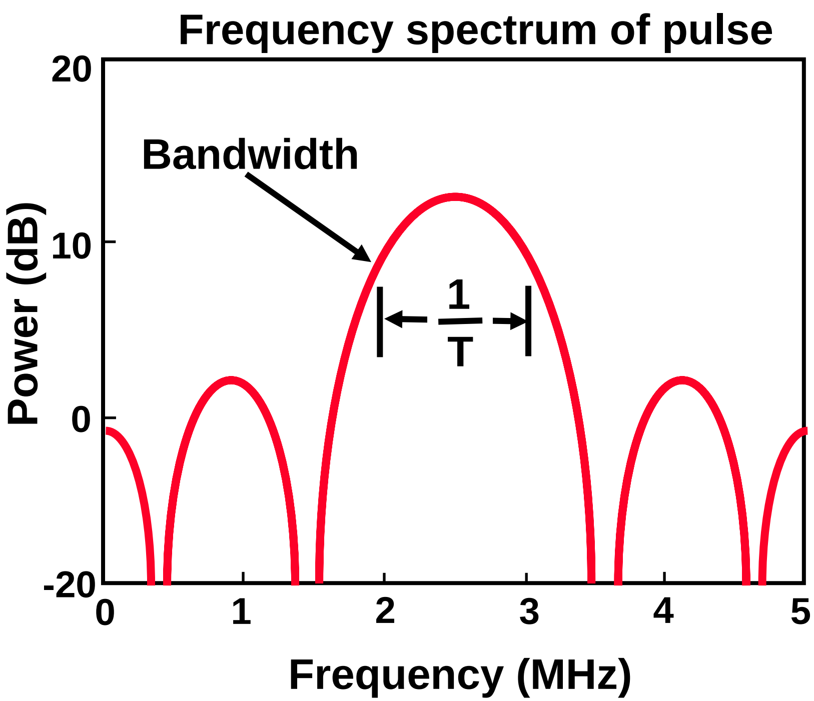
Photo Image
Courtesy of US Air Force

Outline

- **Introduction to radar waveforms and their properties**
  - **Matched filters**
- **Pulse Compression**
  - **Introduction**
  - **Linear frequency modulation (LFM) waveforms**
  - **Phase coded (PC) waveforms**
  - **Other coded waveforms**
- **Summary**

CW Pulse, Its Frequency Spectrum, and Range Resolution





- Range Resolution (Δr)
  - Proportional to pulse width (T)
  - Inversely proportional to bandwidth $(B = 1/T)$

1 MHz Bandwidth => 150 m of range resolution

$$\Delta r = \frac{c\,T}{2}$$

$$\Delta r = \frac{c}{2\,B}$$

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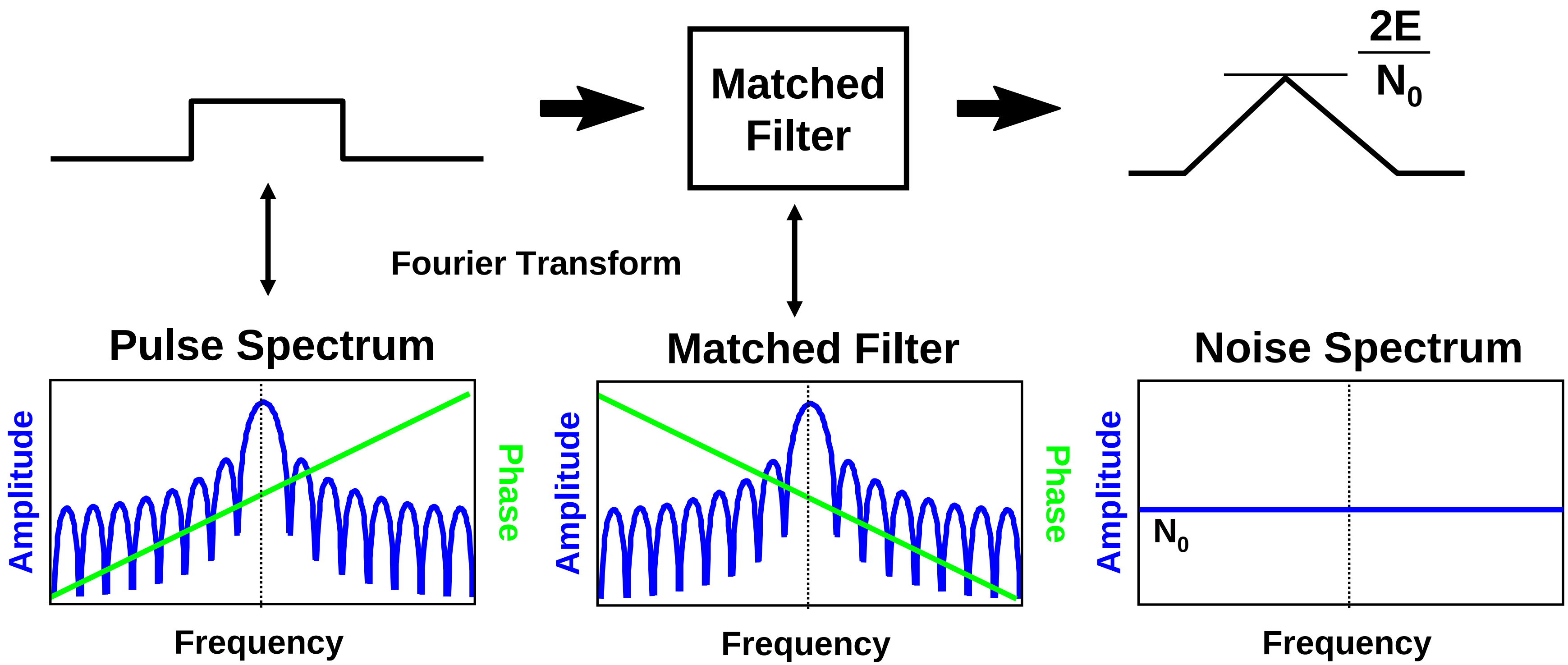
Matched Filter Concept

E = Pulse Energy (Power × Time)



- **Matched Filter maximizes the peak-signal to mean noise ratio**
  - **For rectangular pulse, matched filter is a simple pass band filter**

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Matched Filter Basics

- One wants to pass the received radar echo through a filter, whose output will optimize the Signal-to-Noise Ratio (S/N)
- For white Gaussian noise, the frequency response, $H(f)$, of the matched filter is

Complex conjugate

$$H(f) = A\,S^*(f)\,e^{-2\pi j f t_m}$$

  - The transmitted signal is $s(t)$
  - And $S(f) = \int_{-\infty}^{\infty} s(t)\,e^{-2\pi j f t}\,dt$

- With a little manipulation:
  - Amplitude and phase of Matched Filter are

$$|H(f)| = |S(f)| \qquad \phi_{MF}(f) = -\phi_S(f) + 2\pi f\,t_m$$

Matched Filter Basics (continued)

- **In Chapter 5, Section 2, Skolnik (Reference 1) repeats the classic derivation for the matched filter frequency response for a simple pulse in Gaussian noise**
  - **The interested student can read and follow it readily**

- **It states that the output peak instantaneous* signal to mean noise ratio depends only on ;**
  - **The total energy of the received signal, and**
  - **The noise power per unit bandwidth**

$$\leq \frac{2E}{N}$$

*** The Signal-to Noise ratio used in radar equation calculations is the average signal-to-noise, that differs from the above result by a factor of 2 (half of the above)**

Matched Filters – A Look Forward

- **Note that the previous discussion always assumes that the signal only competes with uniform white Gaussian noise**
- **While for ~80% of a typical radar's coverage this is true, the echoes from the various types of clutter, this is far from true**
  - **Ground, rain, sea, birds, etc**
  - **These different types of backgrounds that the target signal competes with have spectra that are very different from Gaussian noise**
- **The optimum matched filters that need to be used to deal with clutter will be discussed in lectures 12 and 13**

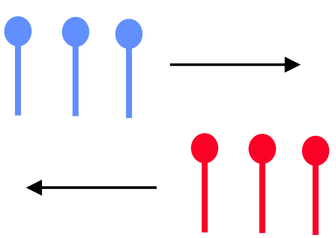
Matched Filter Implementation by Convolution

- Matched filter is implemented by "convolving" the reflected echo with the "time reversed" transmit pulse

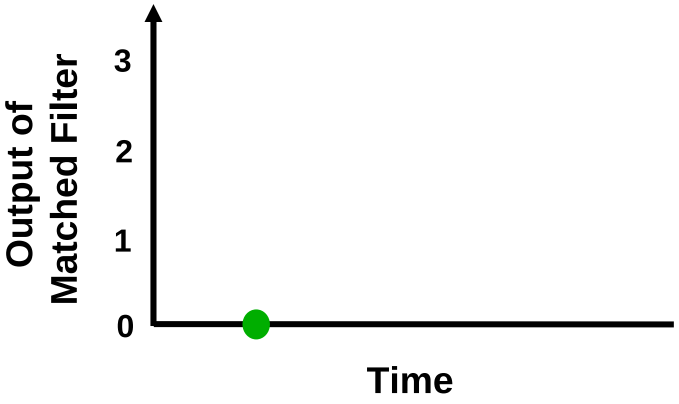
Reflected echo Time reversed pulse

- Convolution process:
  - Move digitized pulses by each other, in steps
  - When data overlaps, multiply samples and sum them up

$$y[k] = \sum_{n=-\infty}^{\infty} h[n]\,x[k-n]$$



No overlap – Output 0



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Implementation of Matched Filter

- **Matched filter is implemented by “convolving” the reflected echo with the “time reversed” transmit pulse**

$x[n]$

$h[-n]$

Reflected echo **Time reversed pulse**

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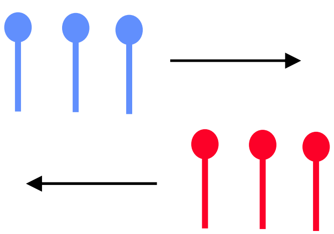

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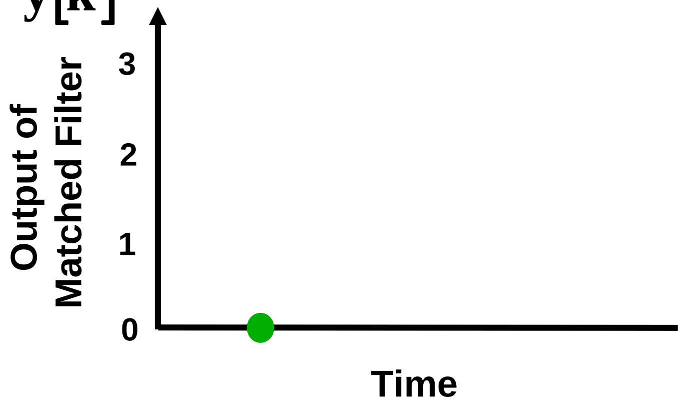

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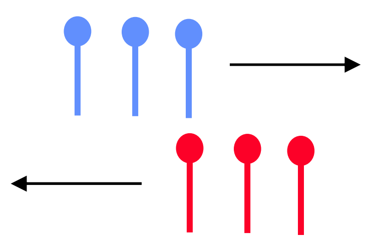
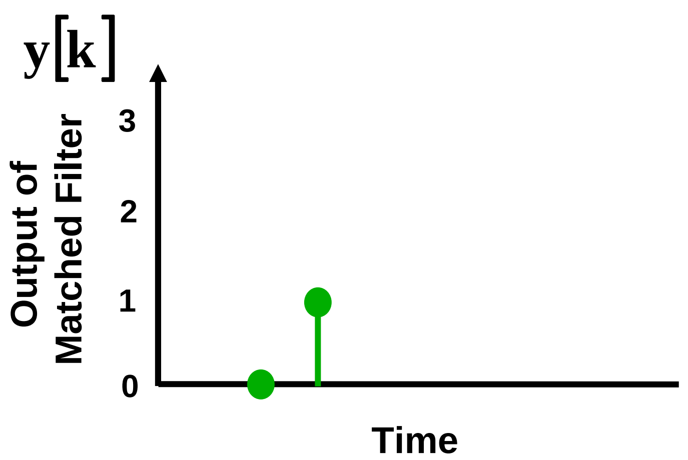


One sample overlaps 1x1 =1



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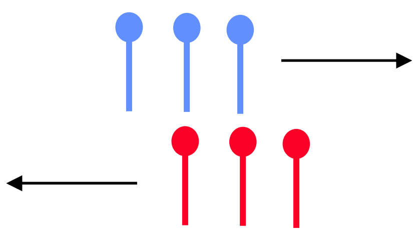
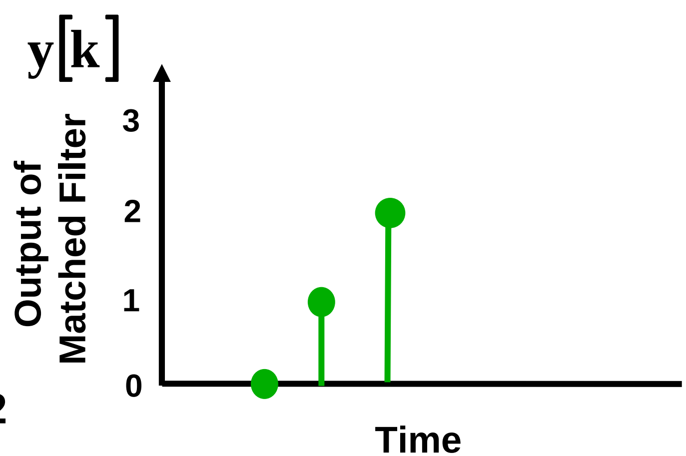


Two samples overlap (1x1) + (1x1) = 2



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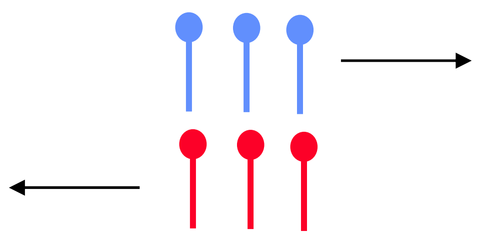

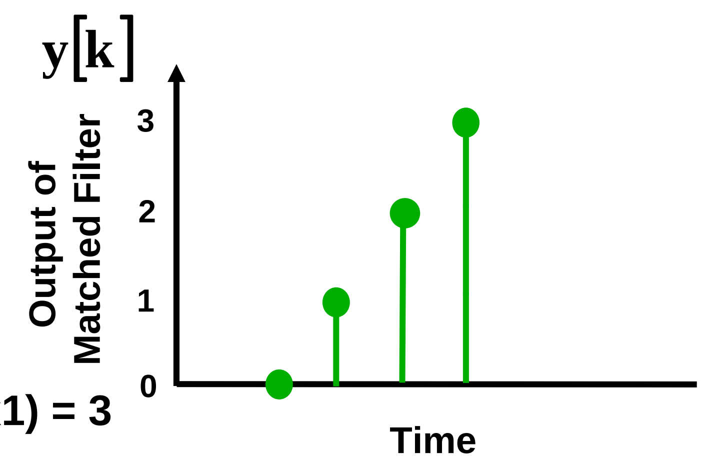

Three samples overlap (1x1) + (1x1) + (1x1) = 3

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Implementation of Matched Filter

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Reflected echo **Time reversed pulse**

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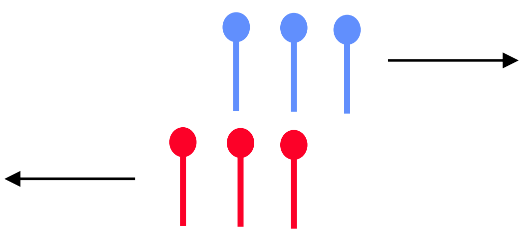

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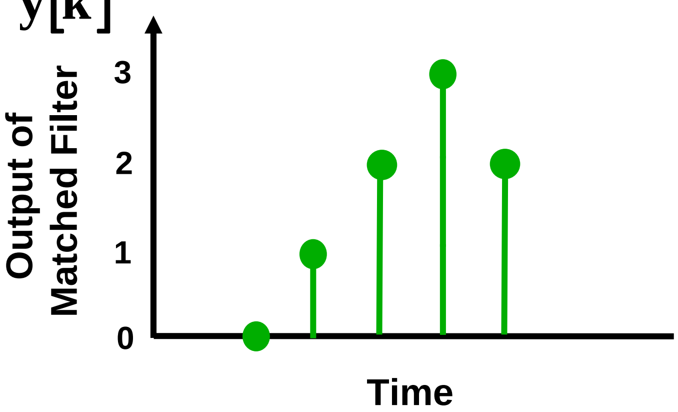

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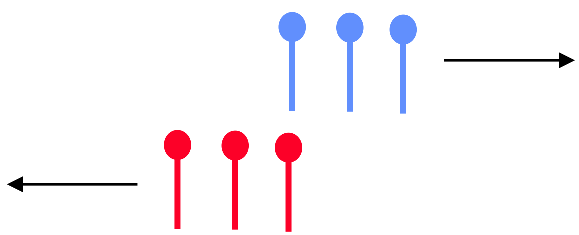

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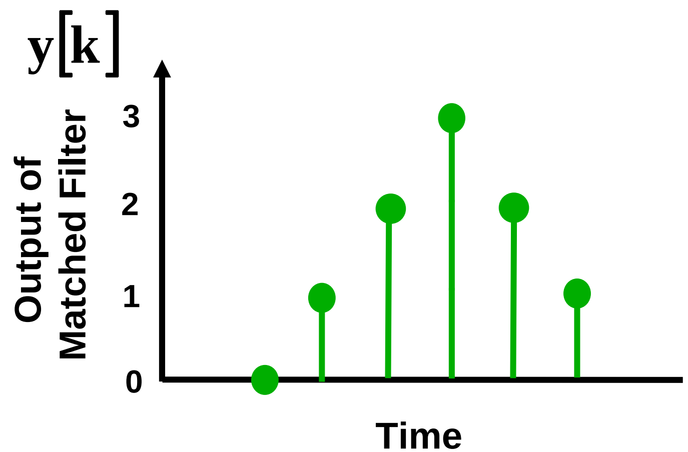

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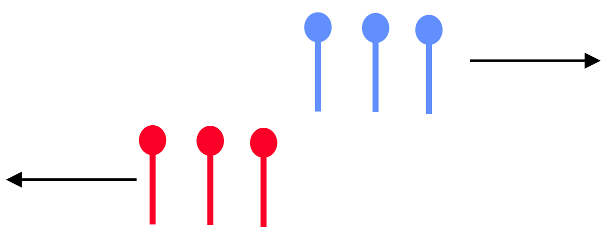

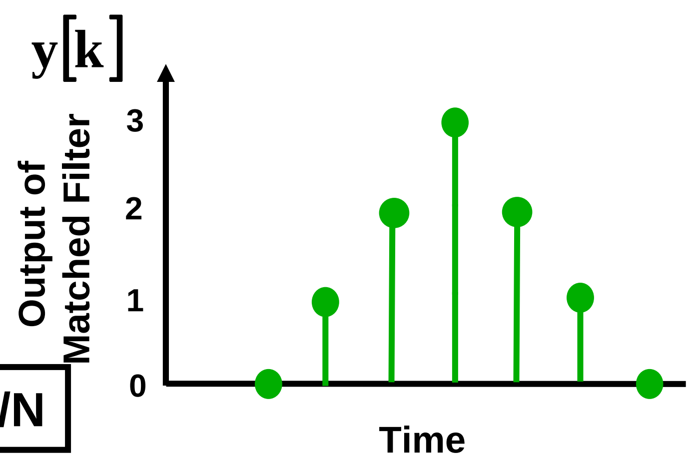

Use of Matched Filter Maximizes S/N

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Motivation for Pulse Compression

- **High range resolution is important for most radars**
  - Target characterization / identification
  - Measurement accuracy
- **High range resolution may be obtained with short pulses**
  - Bandwidth is inversely proportional to pulsewidth
- **Limitations of short pulse radars**
  - High peak power is required for large pulse energy
  - Arcing occurs at high peak power , especially at higher frequencies
    - Example: Typical aircraft surveillance radar
      - 1 megawatt peak power, 1 microsecond pulse, 150 m range resolution, energy in 1 pulse = 1 joule
      - To obtain 15 cm resolution and constrain energy per pulse to 1 joule implies 1 nanosecond pulse and 1 gigawatt of peak power
  - Airborne radars experience breakdown at lower voltages than ground based radars

Motivation for Pulse Compression

- Radars with solid state transmitters are unable to operate at high peak powers
  - The energy comes from long pulses with moderate peak power (20-25% maximum duty cycle)
  - Usually, long pulses, using standard pulsed CW waveforms, result in relatively poor range resolution

- A long pulse can have the same bandwidth (resolution) as a short pulse if it is ***modulated in frequency or phase***

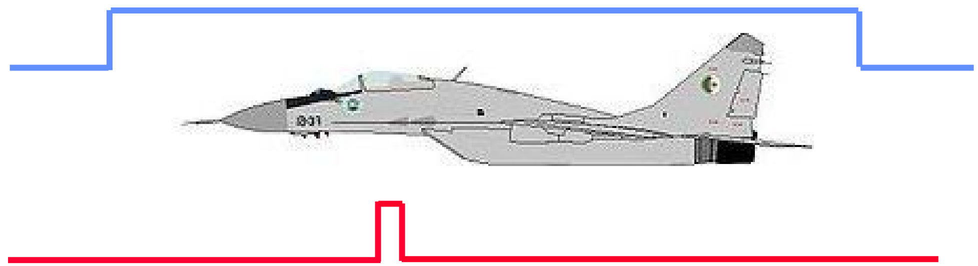
- Pulse compression, using frequency or phase modulation, allows a radar to simultaneously achieve the energy of a long pulse and the resolution of a short pulse

- Two most important classes of pulse compression waveforms
  - Linear frequency modulated (FM) pulses
  - Binary phase coded pulses

Pulse Width, Bandwidth and Resolution for a Square Pulse

Resolution: **Pulse Length is Larger than Target Length**

Cannot Resolve Features Along the Target



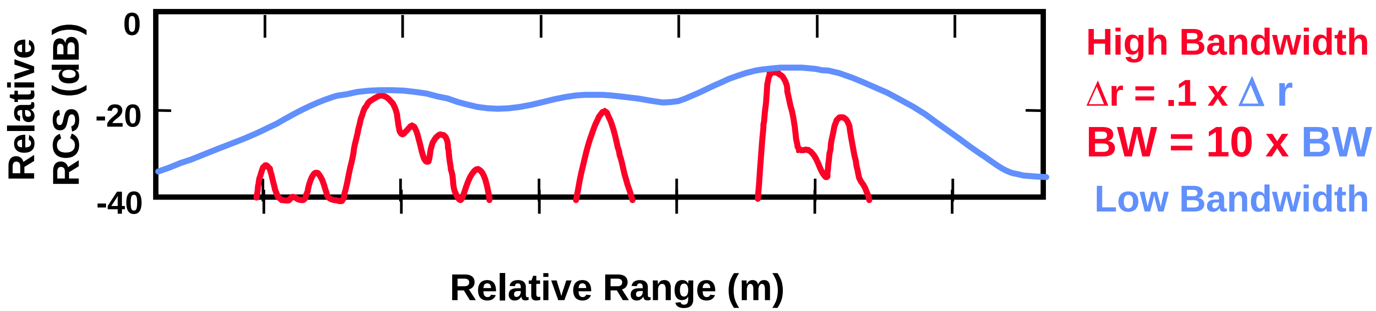
$$\Delta r = \frac{c\,T}{2}$$

$$\Delta r = \frac{c}{2B}$$

Pulse Length is Smaller than Target Length

Can Resolve Features Along the Target

Metaphorical Example :



Shorter Pulses have Higher Bandwidth and Better Resolution

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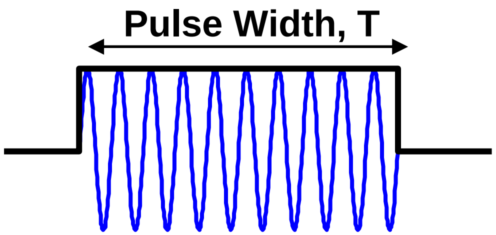
Frequency and Phase Modulation of Pulses

- Resolution of a short pulse can be achieved by modulating a long pulse, increasing the time-bandwidth product
- Signal must be processed on return to "pulse compress"

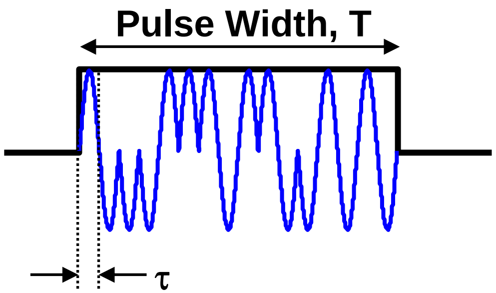
Square Pulse



Bandwidth = 1/T

Time × Bandwidth = 1

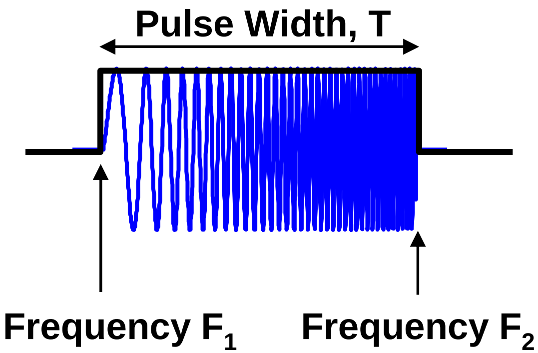
Binary Phase Coded Waveform



Bandwidth = $1/\tau$

Time × Bandwidth = T/τ

Linear Frequency Modulated Waveform



Bandwidth = $\Delta F = F_2 - F_1$

Time × Bandwidth = $T\Delta F$

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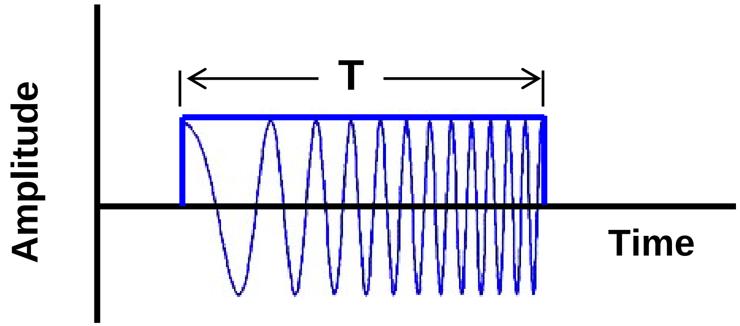
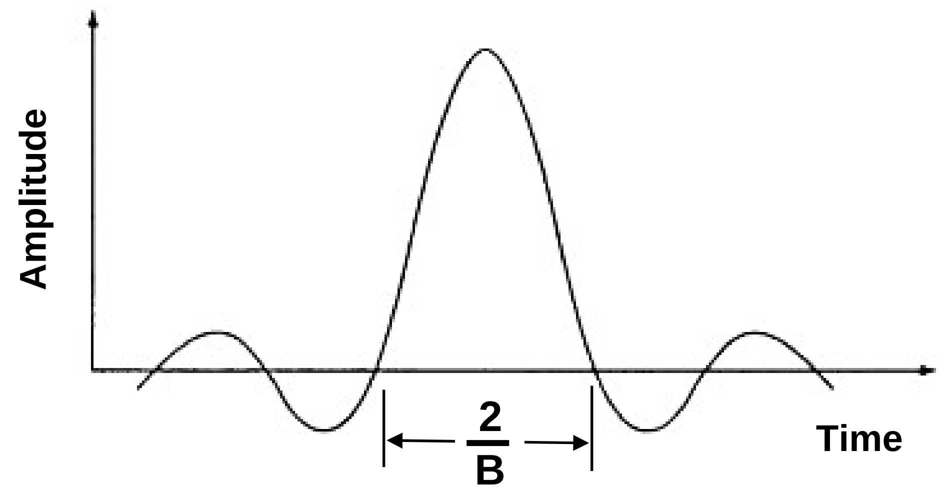
Linear FM Pulse Compression

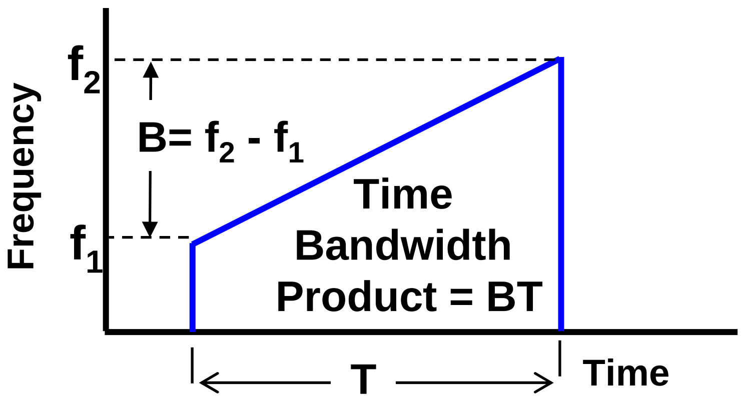
Linear FM waveform **Increasing Frequency**



Output of Pulse Compression Filter



Frequency of transmitted pulse as a function of time



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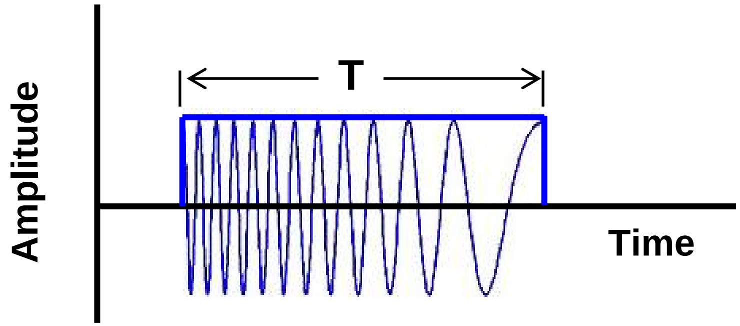
Linear FM Pulse Compression

Linear FM waveform **Decreasing Frequency**



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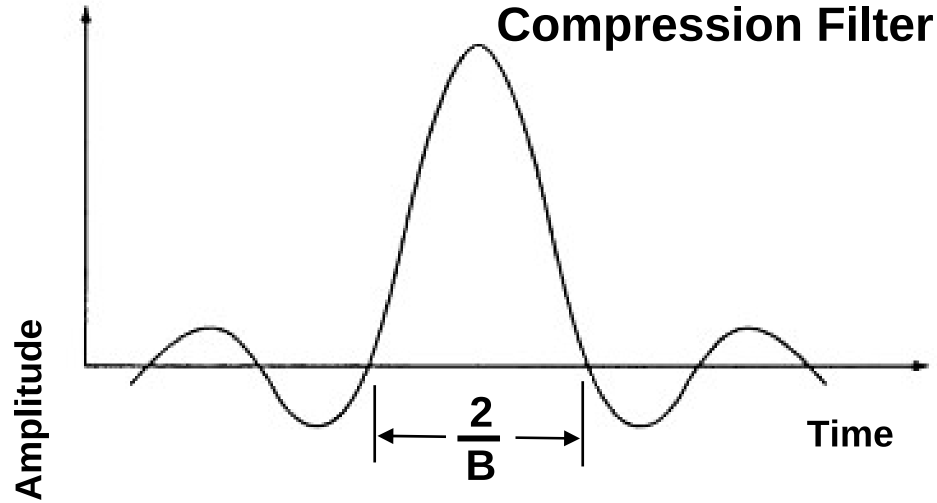

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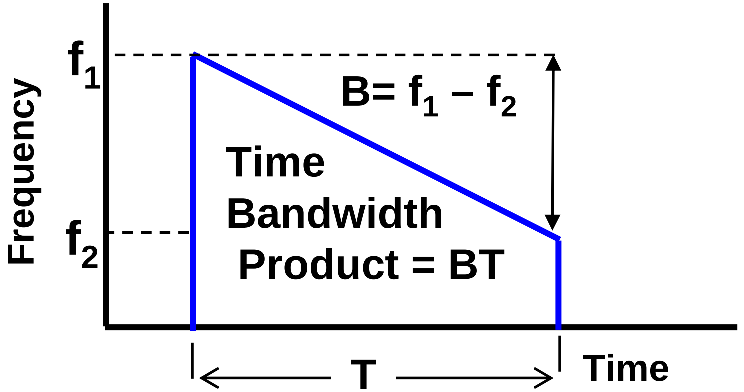

Because range is measured by a shift in Doppler frequency, there is a coupling of the range and Doppler velocity measurement

Viewgraph courtesy of MIT Lincoln Laboratory
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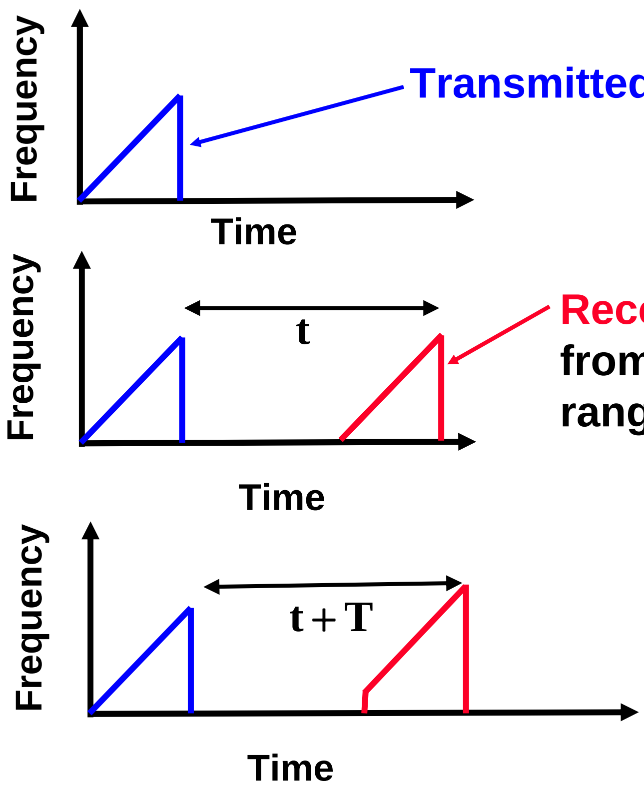
Range Doppler Coupling with FM Waveforms

Frequency vs. Time



Transmitted Waveform

Waveform Slope = $\frac{B}{T}$

Received Waveform from a stationary target at range $R = ct/2$

Is the **Received Waveform** from a stationary target at range $R_1 = c(t + T)/2$ or from a moving target at $R = ct/2$, with Doppler frequency, $f_D = Bt/T$

Range and Doppler measurements are coupled with Frequency modulated waveforms

Linear FM Pulse Compression Filters

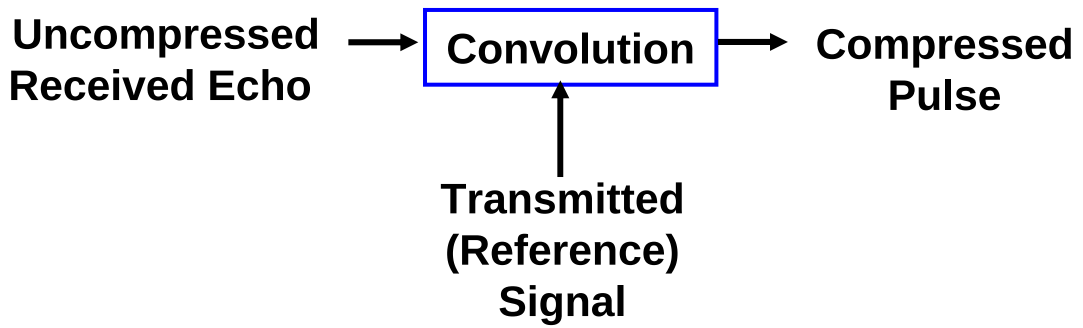
- **Linear FM pulse compression filters are usually implemented digitally**
  - **A / D converters can often provide the very wide bandwidths required of high resolution digital pulse compression radar**
- **Two classes of Linear FM waveforms**
  - **Narrowband Pulse Compression**
  - **High Bandwidth Pulse Compression (aka “Stretch Processing”)**

Linear FM Pulse Compression by Digital Processing

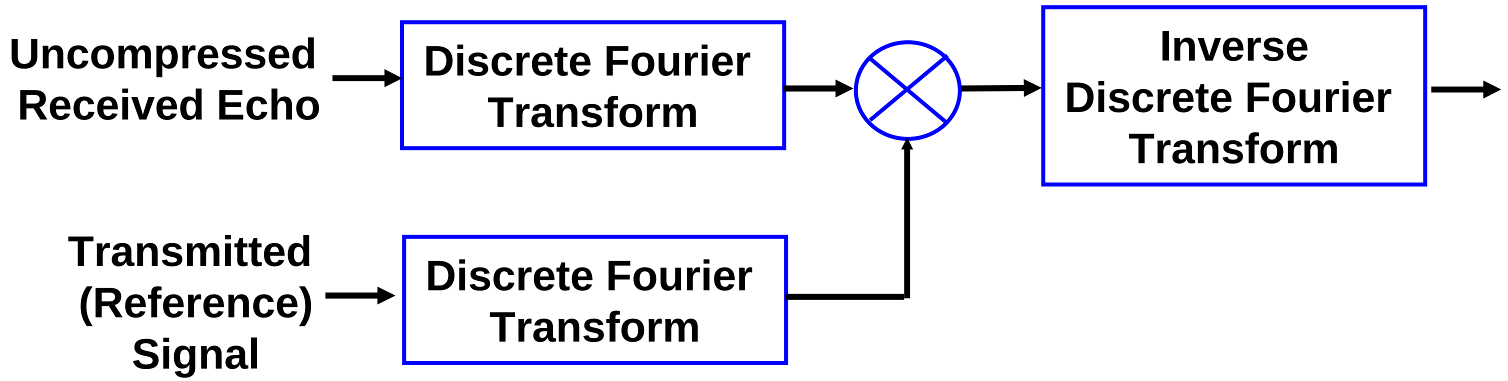
- Linear FM pulse compression waveforms can be processed and generated at low power levels by digital methods, when A / D converters are available with the required bandwidth and number of bits
- Digital methods are stable and can handle long duration waveforms
- The same basic digital implementation can be used with :
  - multiple bandwidths
  - multiple pulse durations
  - different types of pulse compression modulation
  - good phase repeatability
  - low time sidelobes
  - when flexibility is desired in waveform selection

Implementation Methods for LFM Pulse Compression

- **Direct Convolution in Time Domain**



- **Frequency Domain Implementation**



Reduction of Time (Range) Sidelobes

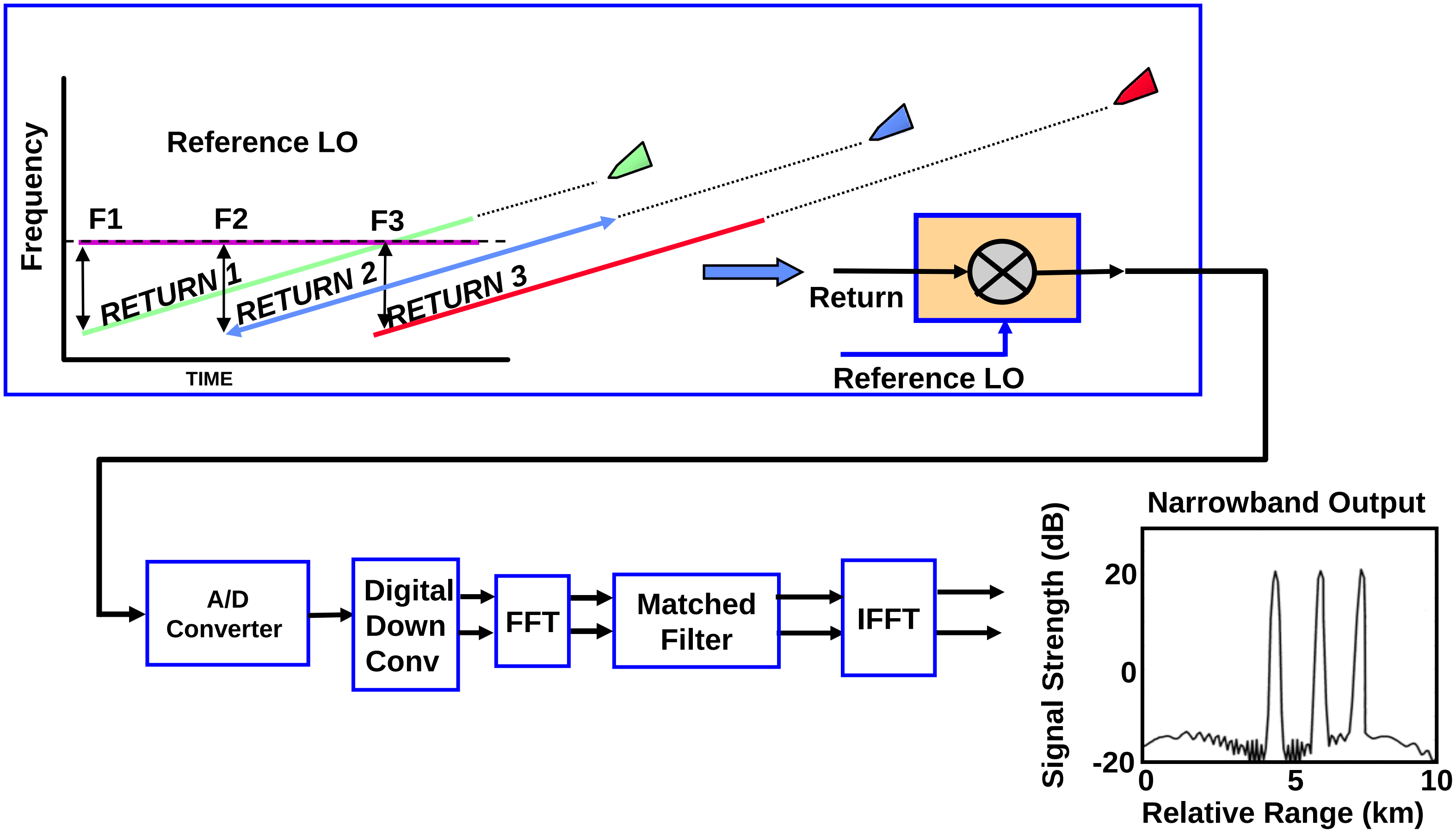
- Optimum (matched filter) output has sin(x) / x form
  - 13.2 db time (range) sidelobe
  - High sidelobes can be mistaken for weak nearby targets

- Potential solution - Amplitude taper on transmit
  - Klystrons, TWTs and CFAs operate in saturation
  - Solid state transmitters can, but most often don’t have this capability
    - Higher efficiency
    - Seldom done

- Time sidelobes of linear FM waveforms are usually reduced by applying an amplitude weighting on the receive pulse
  - Typical Results
    - Mismatch loss of about 1 dB
    - Peak sidelobe reduced to 30 dB

Narrowband Pulse Compression

- Used for NB waveforms
  - Receive LFM wide pulse
    - Wide pulsewidth for good detection
  - Process signal to narrow band - pulse range resolution



Wideband Stretch Processing - Overview

- In many cases involving high bandwidth radar systems, the instantaneous bandwidth of the linear FM waveform is greater than the sampling rates of available A/D converter technology

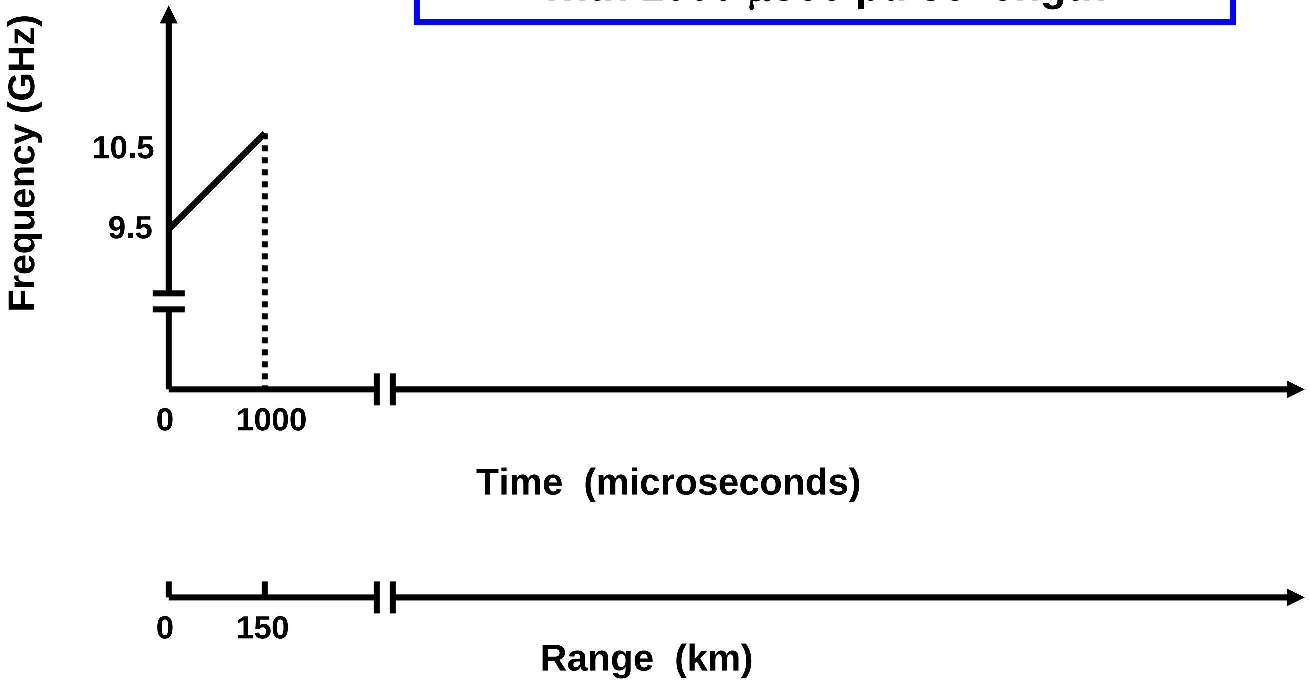
- In these cases, "Stretch Processing*", can be employed to yield high range resolution (commensurate with that very high bandwidth) over a limited range window by processing the data in a manner that makes use of the unique range-Doppler coupling of linear FM waveforms

- This technique will be now described in more detail.

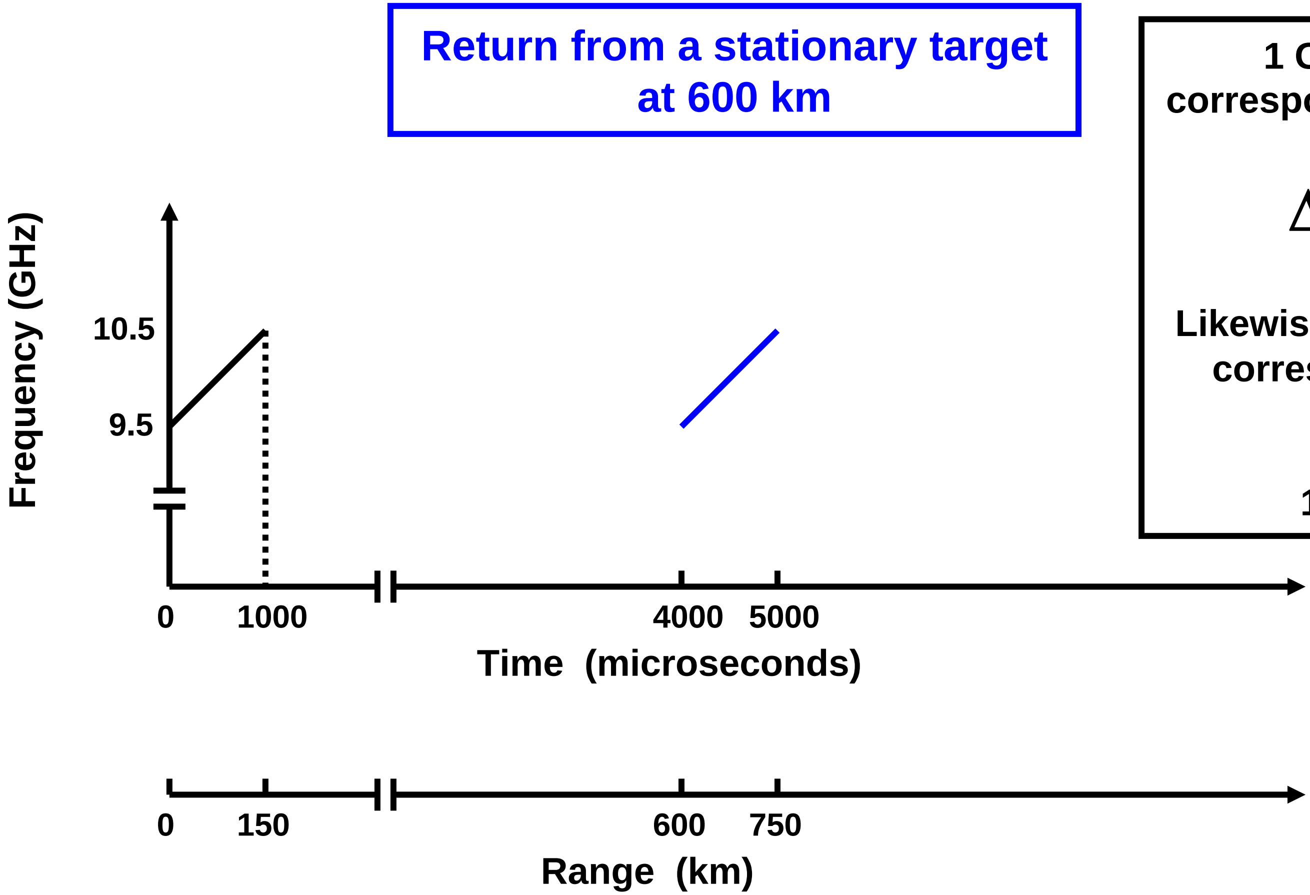
*Note: Dr. W. Caputi was awarded the IEEE Dennis Picard Medal in 2005 in recognition of his development of this technique and other significant achievements

Stretch Processing Example

Transmit a 1 GHz Bandwidth Wideband Linear FM Pulse at X-Band with 1000 μsec pulse length



Example of Stretch Processing



1 GHz in 1000 μsec corresponds to a range of 150 km

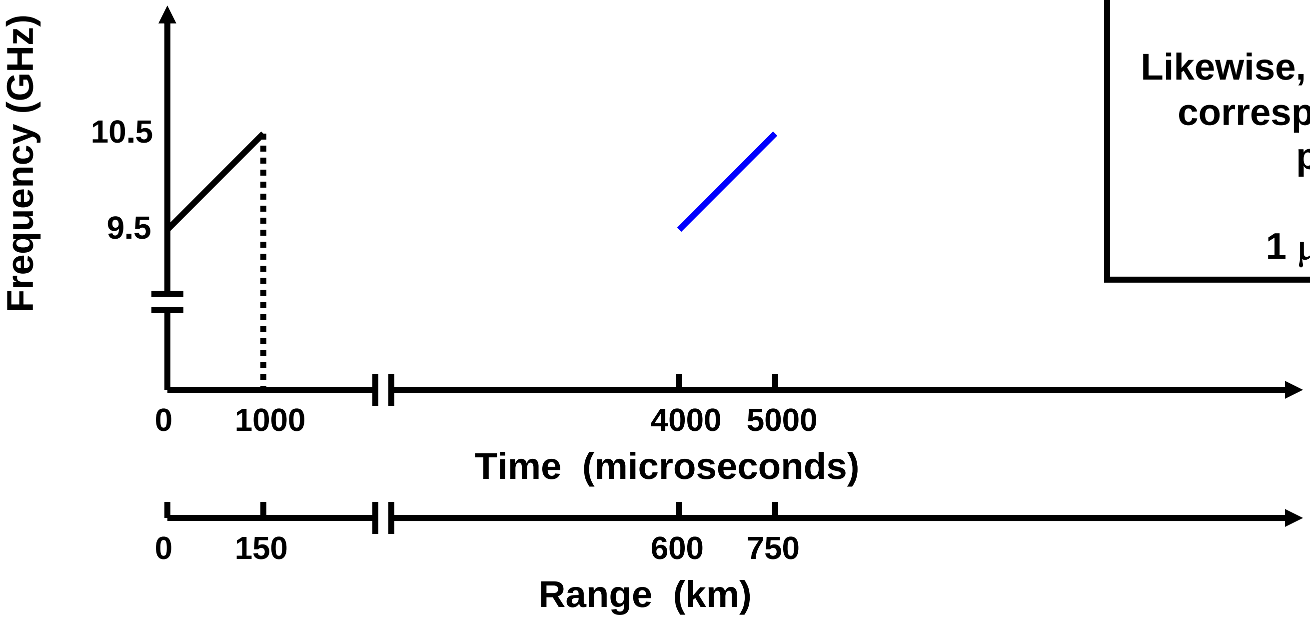
$$\Delta r = c \, \Delta t / 2$$

Likewise, 600 m range extent corresponds to a (4 μsec pulse delay

1 μsec = 150 m

Example of Stretch Processing

Return from 2 stationary targets at 600 km and 600.006 km



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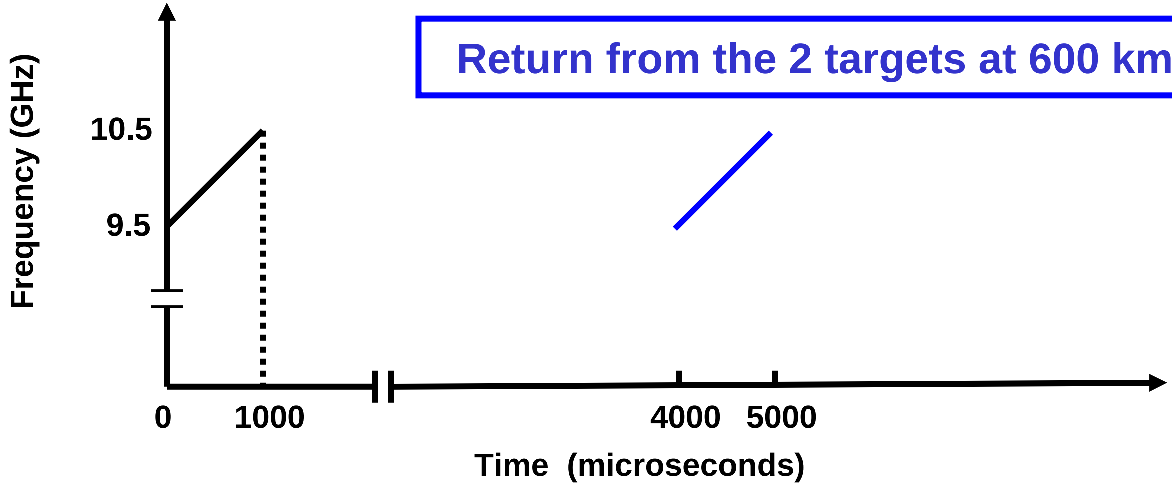
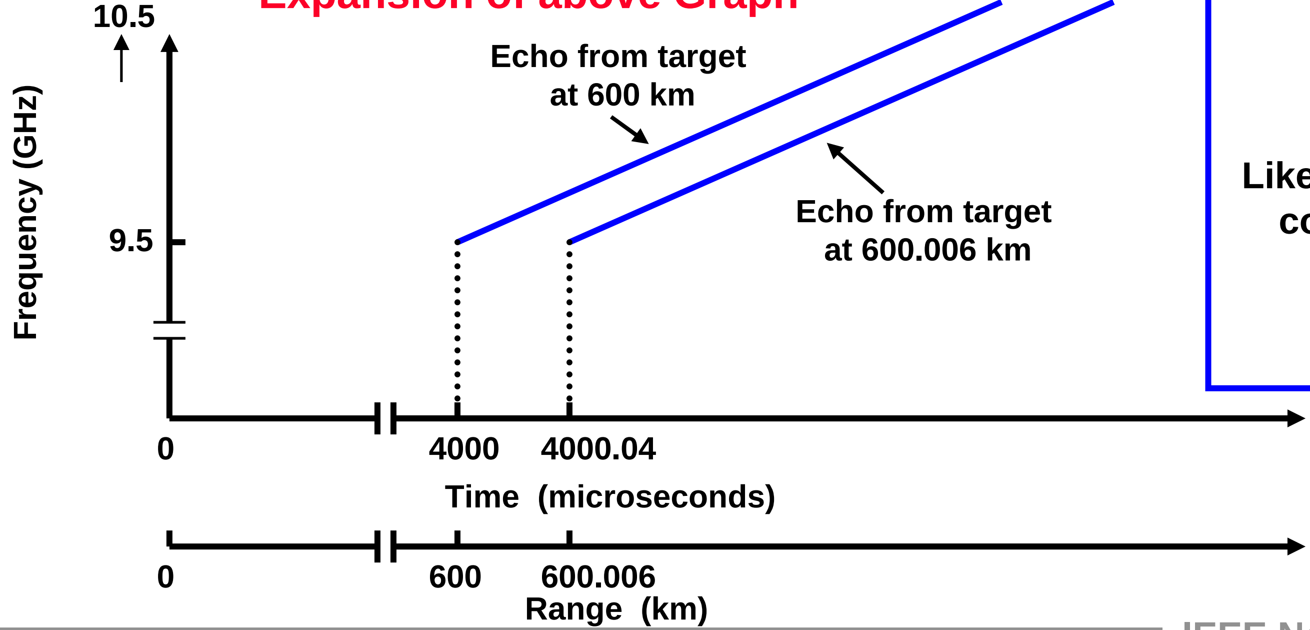


Expansion of above Graph



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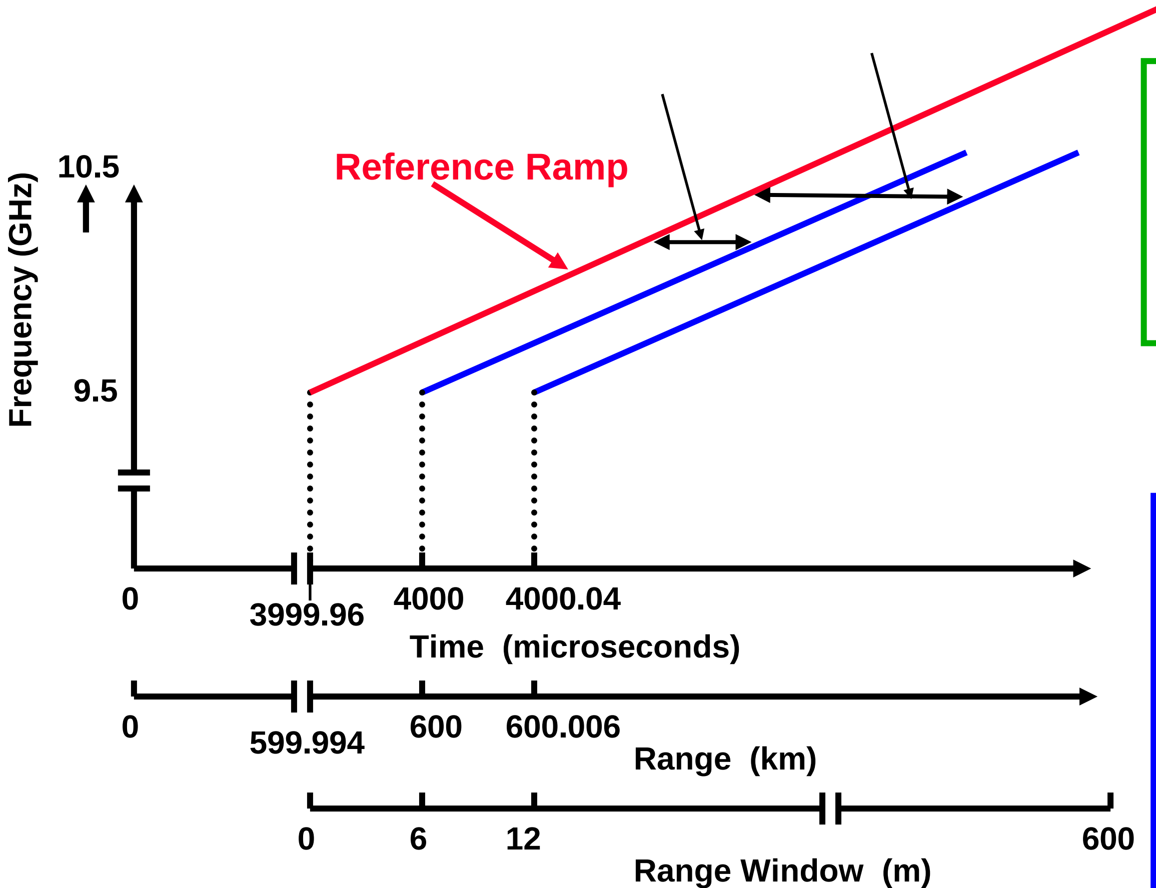
1 μsec = 150 m

Example of Stretch Processing



Mix Radar Echo Signal with a Linear FM Reference Ramp Having Same Slope as Transmitted Pulse

1000 GHz in 1000 μsec corresponds to a range of 150 km

$$\Delta r = c\,\Delta t / 2$$

Likewise, 600 m range extent corresponds to a (4 μsec pulse delay

1 μsec = 150 m

Return from 2 targets at 600 km and 600.006 km

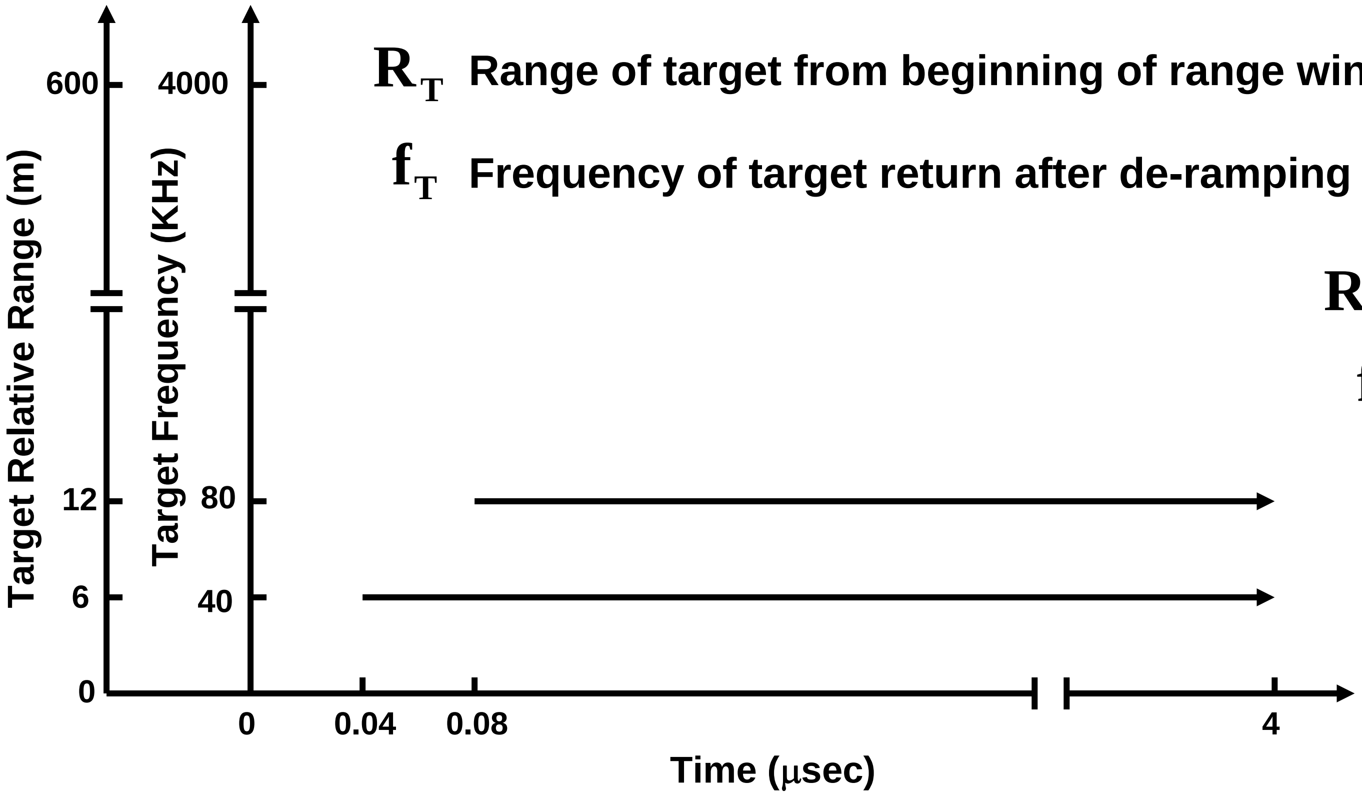
Example of Stretch Processing

R_T Range of target from beginning of range window

f_T Frequency of target return after de-ramping

$$R_T = c/2 f_T$$

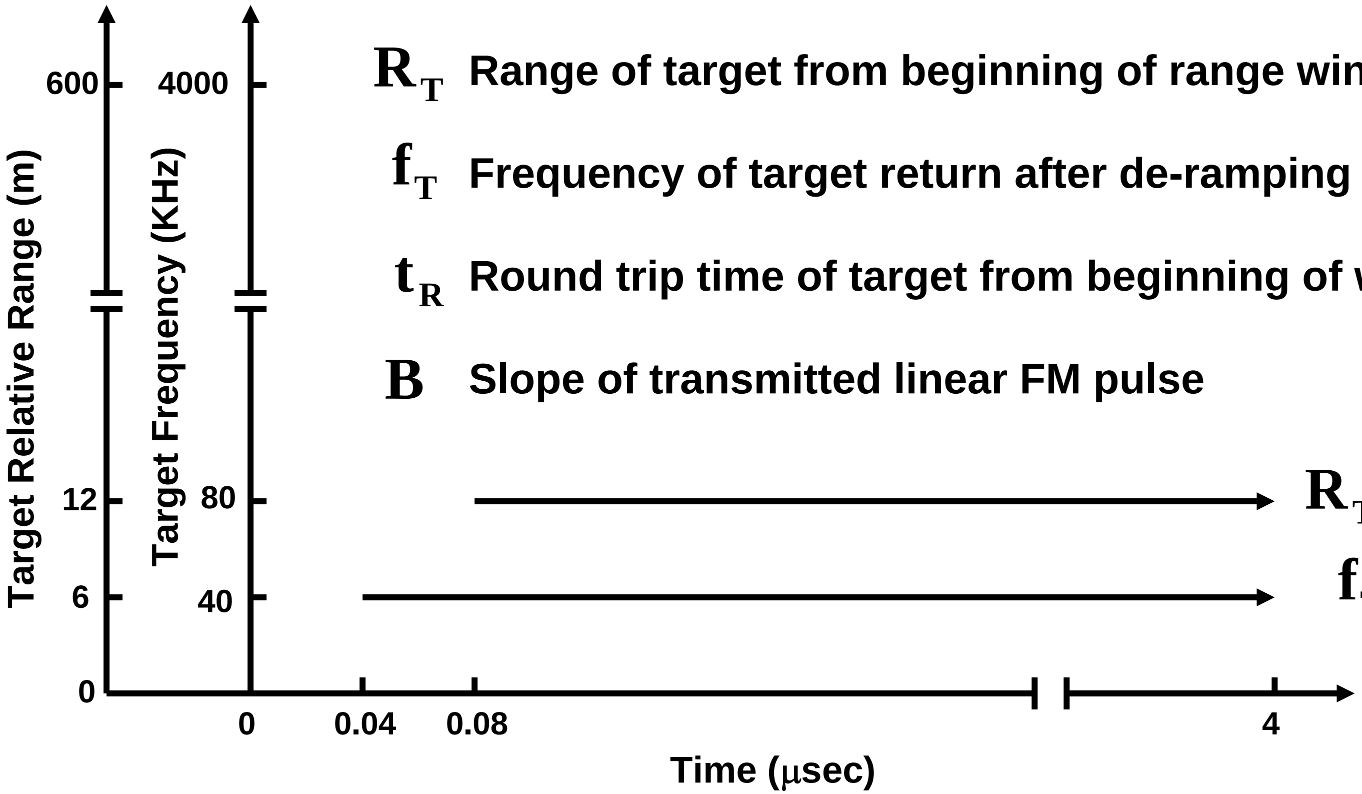
$$f_T = B\, t_R$$



The separation in distance of the two targets corresponds to a time delay through $\Delta R = c\, \Delta t / 2$

The relative time delay is related to is related to the above target frequencies through the slope of the FM waveform

Example of Stretch Processing



R_T Range of target from beginning of range window

f_T Frequency of target return after de-ramping

t_R Round trip time of target from beginning of window

B Slope of transmitted linear FM pulse

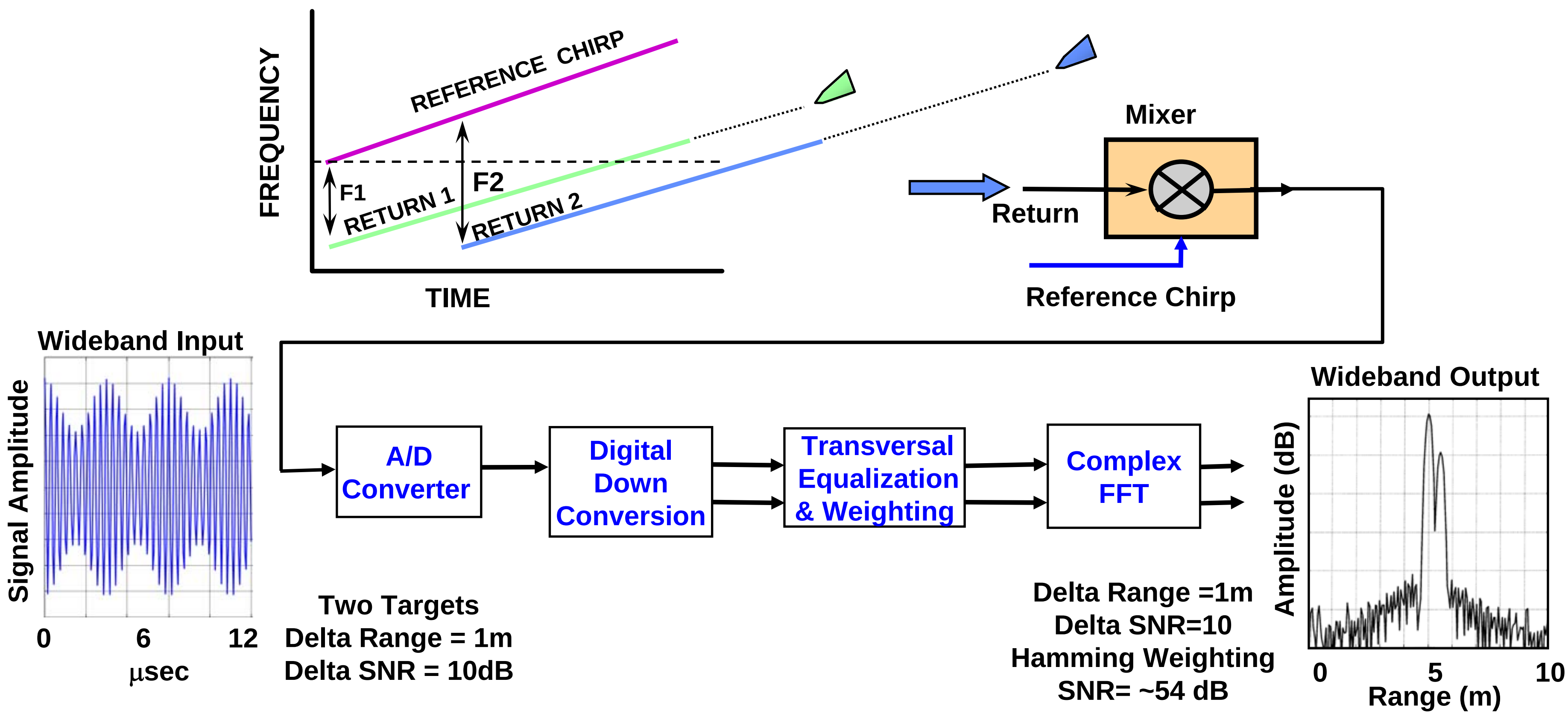
$R_T = c / 2 f_T$

$f_T = B\, t_R$

The separation in distance of the two targets corresponds to a time delay through $\Delta R = c\, \Delta t / 2$
The relative time delay is related to is related to the above target frequencies through the slope of the FM waveform

Implementation of Stretch Processing

- Used for all wide bandwidth waveforms
  - Receive waveform mixed with similar reference waveform prior to A/D conversion
  - Frequency representation of resulting sinusoids translates into range of targets



Linear FM - Summary

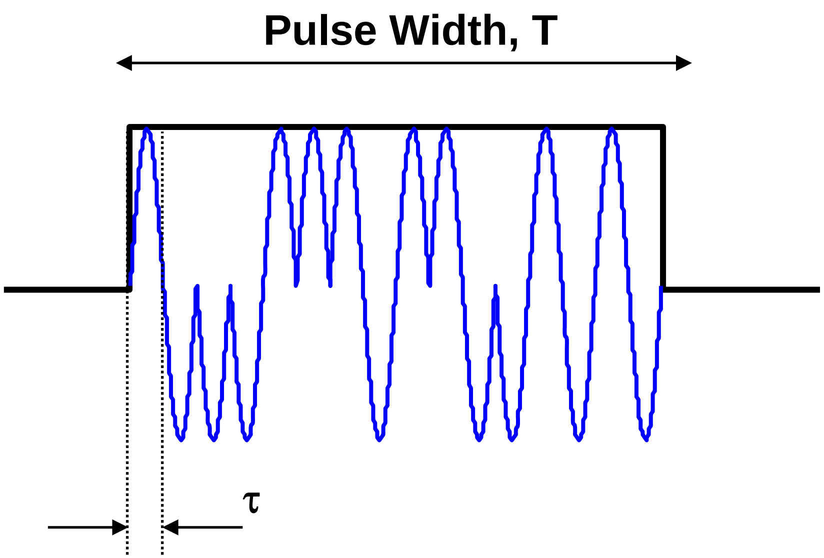
- **Waveform used most often for pulse compression**
- **Less complex than other methods**
  - **Especially if stretch processing is not appropriate**
- **Weighting on receive usually required**
  - **-13.2 dB to -30 dB sidelobes with 1 dB loss**
- **Range Doppler coupling**
  - **Sometimes of little consequence**

Outline

- Introduction to radar waveforms and their properties
  - Matched filters
- Pulse Compression
  - Introduction
  - Linear frequency modulation (LFM) waveforms
  - Phase coded (PC) waveforms
  - Other coded waveforms
- Summary

Binary Phase Coded Waveforms

Binary Phase
Coded Waveform



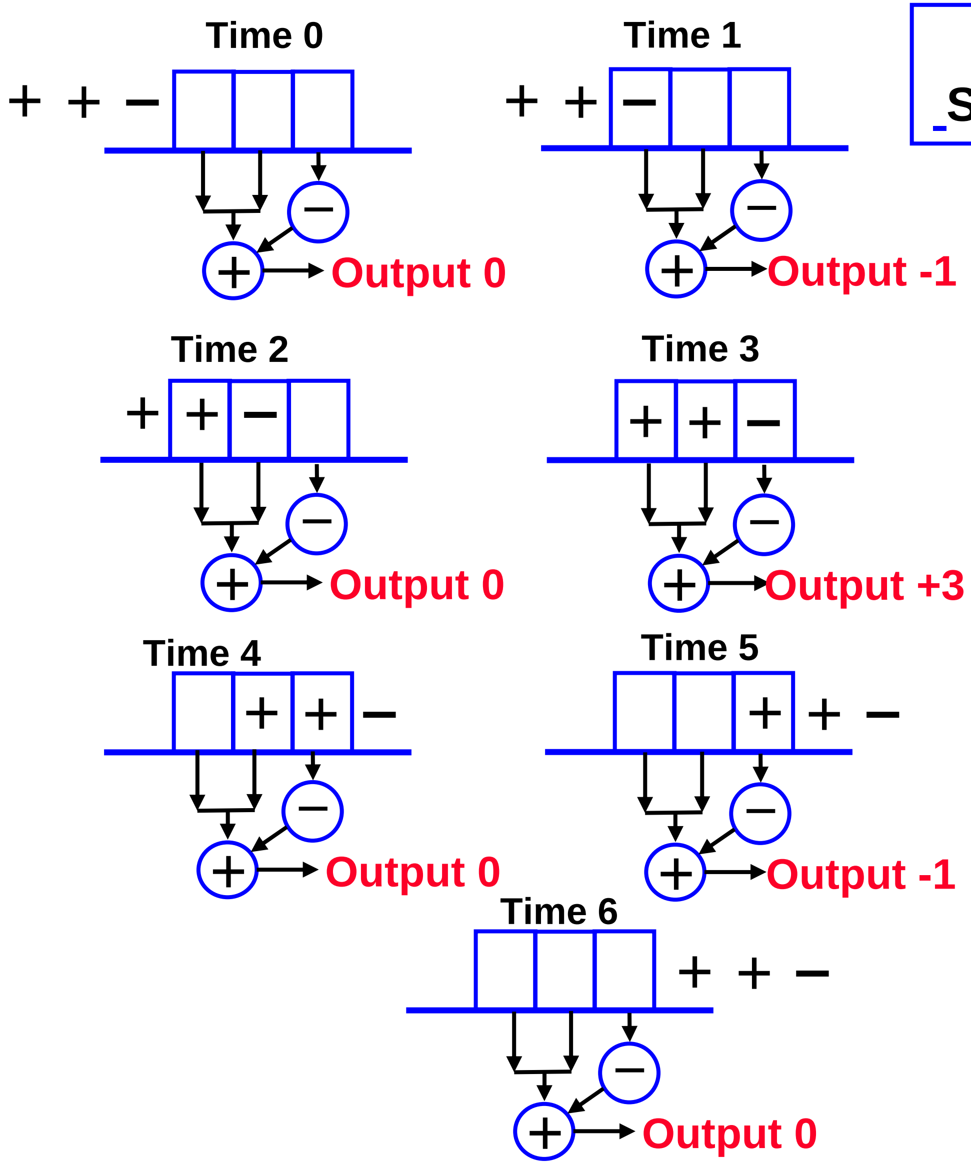
Bandwidth = $1/\tau$

Pulse Compression Ratio = T/τ

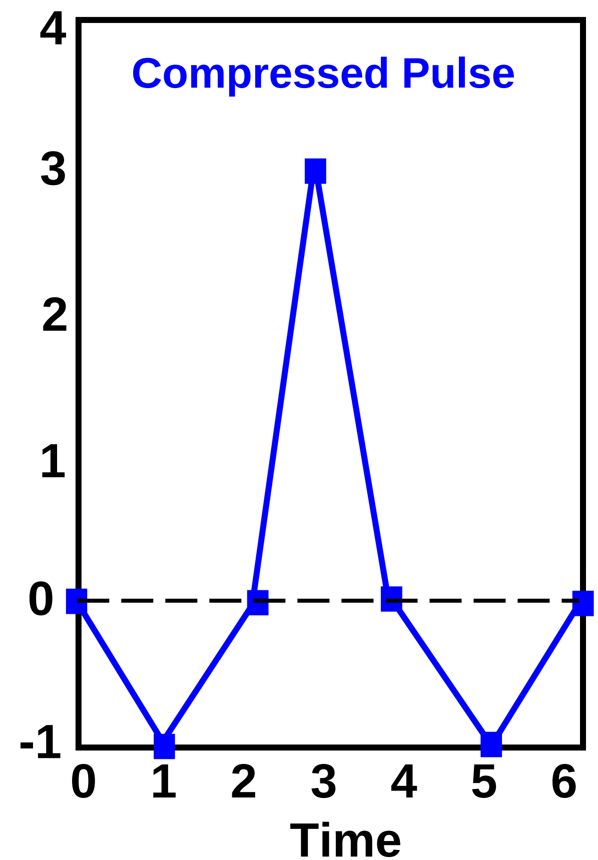
- Changes in phase can be used to increase the signal bandwidth of a long pulse
- A pulse of duration T is divided into N sub-pulses of duration τ
- The phase of each sub-pulse is changed or not changed, according to a binary phase code
- Phase changes 0 or π radians (+ or -)
- Pulse compression filter output will be a compressed pulse of width τ and a peak N times that of the uncompressed pulse

Matched Filter - Binary Phase Coded Pulse

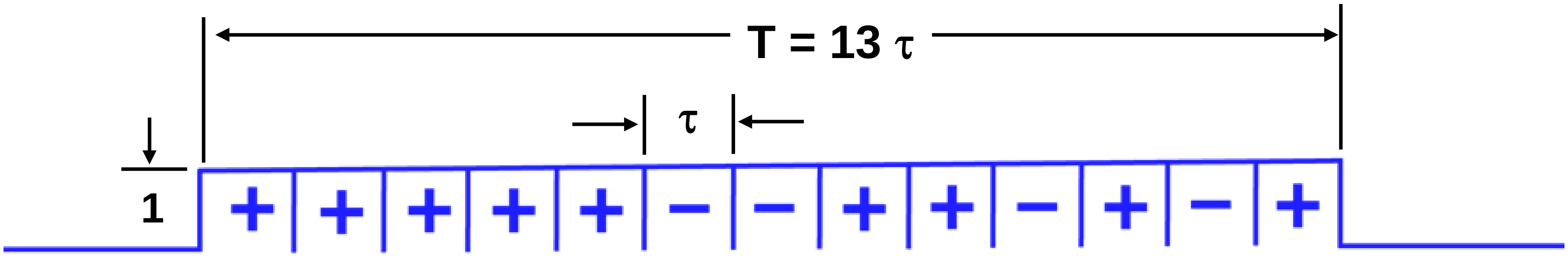
Example - 3 Bit Barker Code
Seven Time Steps of Delay Line



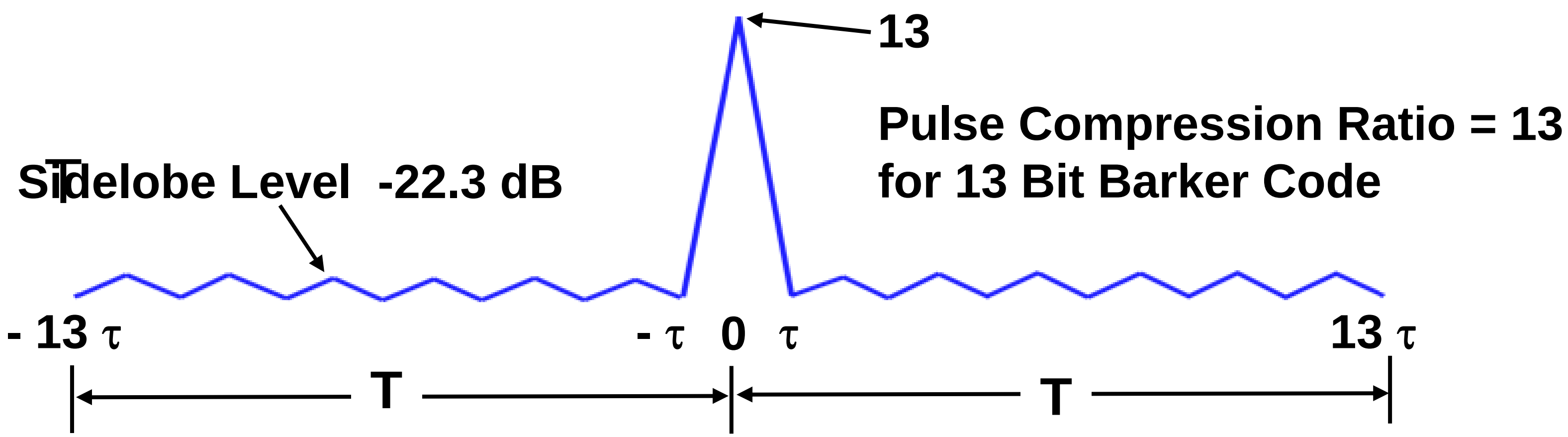
Matched Filter Output



Example - 13 Bit Barker Code



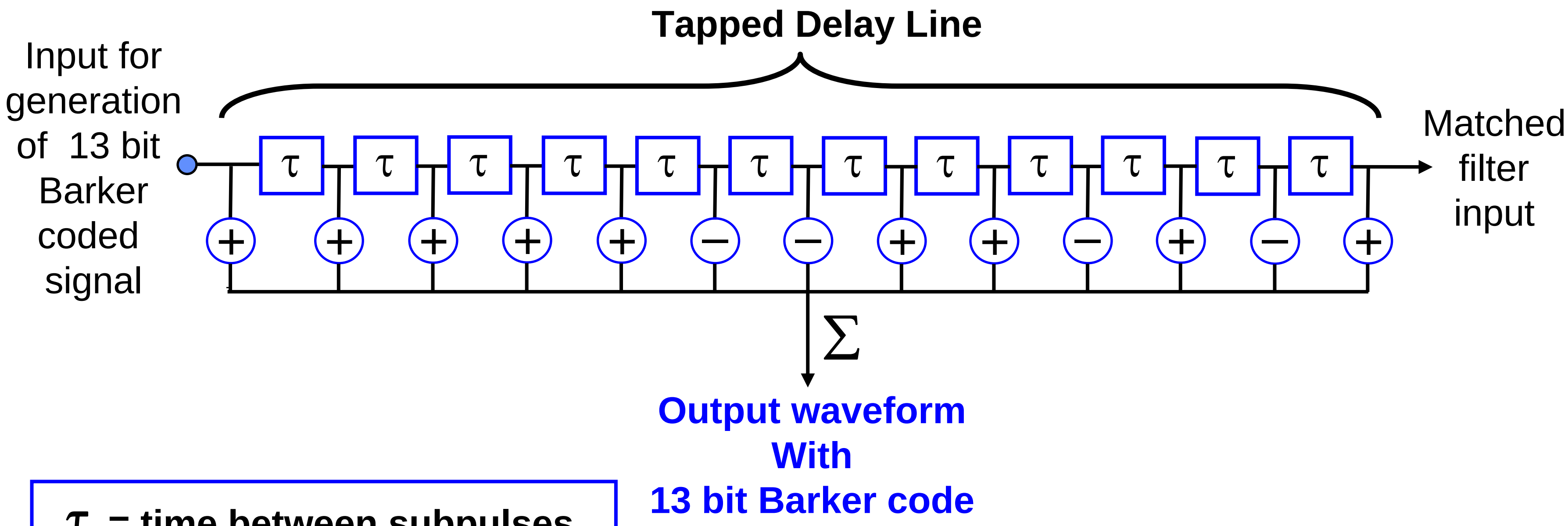
A long pulse with 13 equal sub-pulses, whose individual phases are either 0 (+) or π (-) relative to the un-coded pulse



Auto-correlation function of above pulse, which represents the output of the matched filter

Tapped Delay Line

Generating the Barker Code of Length 13



τ = time between subpulses

T = 13 τ = total pulse length

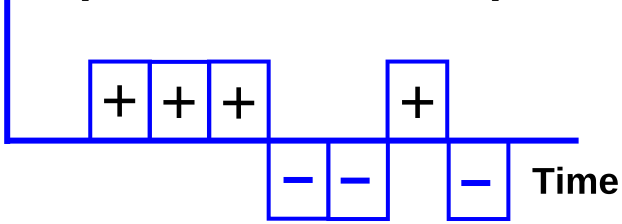
Barker Codes

Code Length	Code Elements	Sidelobe Level (dB)
2	+ - , + +	- 6.0
3	+ + -	- 9.5
4	+ + - + , + + + -	- 12.0
5	+ + + - +	- 14.0
7	+ + + - - + -	- 16.9
11	+ + + - - - + - - + -	- 20.8
13	+ + + + + - - + + - + - +	- 22.3

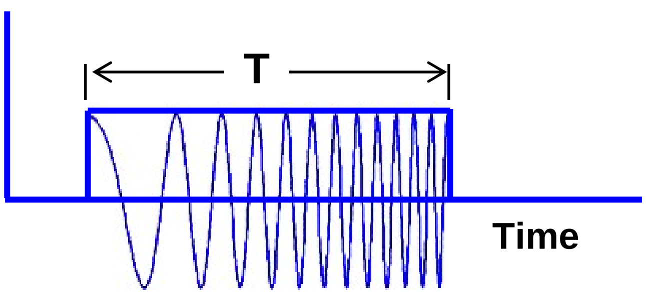
- The 0, and π binary phase codes that result in equal time sidelobes are called Barker Codes
- Sidelobe level of Barker Code is $1 / N^2$ that of the peak power (N = code length)
- None greater than length 13

Range Sidelobe Comparison

Binary Phase Coded Waveform (7 bit Barker code)

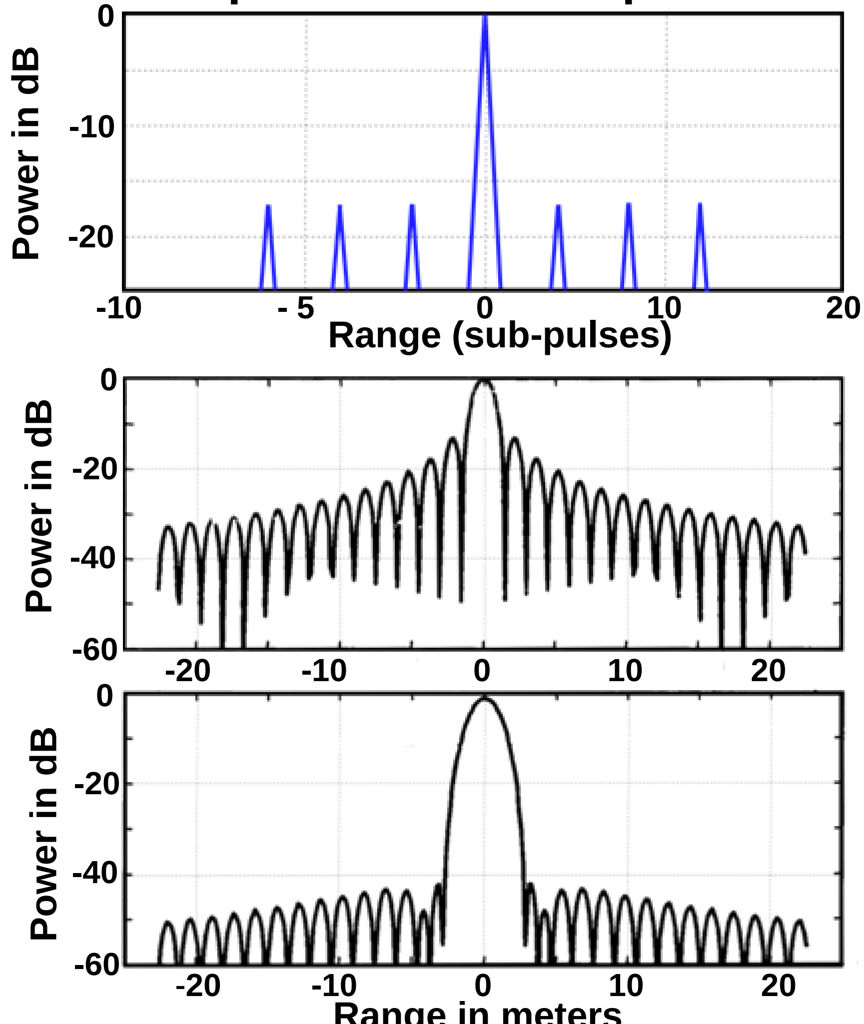


Linear FM Waveform (unweighted)



Linear FM Waveform (Hamming sidelobe weighting))

Output of Pulse Compression



Outline

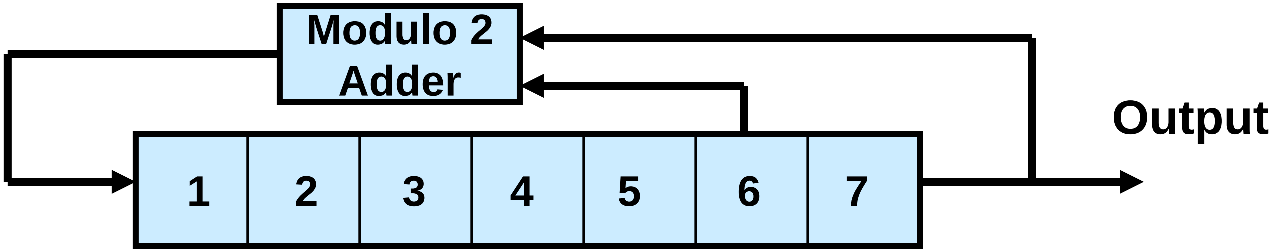
- **Introduction to radar waveforms and their properties**
  - **Matched filters**

- **Pulse Compression**
  - **Introduction**
  - **Linear frequency modulation (LFM) waveforms**
  - **Phase coded (PC) waveforms**
  - **Other coded waveforms**
    - **Linear recursive sequences**
    - **Quadriphase codes**
    - **Polyphase codes**
    - **Costas Codes**

- **Summary**

Linear Recursive Sequences (Shift Register Codes)

- Used for N >13
- Shift register with feedback & modulo 2 arithmetic which generates pseudo random sequence of 1s & 0s of length 2^N-1
  - – N = number of stages in shift register
  - – Also called :
    - Linear recursive sequence (code)
    - Pseudo-random noise sequence (code)
    - Pseudo-noise (PN) sequence (code)
    - Binary shift register sequence (code)
- Different feedback paths and initial settings yield different different sequences with different sidelobe levels
- Example 7 bit shift register for generating a pseudo random linear recursive sequence, N = 127 and 24 dB sidelobes



Quadriphase Codes

- Used to alleviate some of the problems of binary phase codes
  - Poor fall off of radiated pattern
  - Mismatch loss in the receiver pulse compression filter
  - Loss due to range sampling when pulse compression is digital
- Description of Quadriphase codes
  - Obtained by operating on binary phase codes with an operator
  - 0, $\pi/2$, π, or $3\pi/2$
  - Between subpulses the phase change is $\pi/2$
  - Each subpulse has a 1/2 cosine shape
    - Rather than rectangular
  - Range straddling losses are reduced

Polyphase (Frank) Codes

- Phase quantization is less than π radians
- Produces lower range sidelobes than binary phase coding
- Tolerant to Doppler frequency shifts
  - If Doppler frequencies are not too large

M x M Matrix Defining
Frank Polyphase Code

0 0 0 0 . . . 0
0 1 2 3 . . . (N-1)
0 2 4 6 . . . 2(N-1)
0 3 6 9 . . . 3(N-1)
.
.
0 (N-1). . . $(N-1)^2$

Example of Frank Matrix with M = 5
Pulse Compression Ratio N = M x M = 25
Peak sidelobe 23.9 dB
Basic phase increment $2\pi/5$ = 72 degrees

0	0	0	0	0
0	72	144	216	288
0	144	288	72	216
0	216	72	288	144
0	288	216	144	72

The phases of each of the M^2 subpulses are found by starting at the upper left of the matrix and reading each row in succession from left to right. Phases are modulo 360 degrees

Costas Codes

- Frequencies in the subpulse are changed in a prescribed manner
- A pulse of length T is divided into M contiguous subpulses
- The frequency of each subpulse is selected from M contiguous frequencies
- The frequencies are separated by the reciprocal of the subpulse, $\Delta B = M/T$
  - There are B / M different frequencies
  - The width of each subpulse is T / M
  - The pulse compression ratio is $B\,T = M^2$
- Costas developed a method of selection which minimizes the range and Doppler sidelobe levels

Other Coded Waveforms

- **These are some of the other methods of phase and frequency coding radar waveforms.**
  - **They are covered in the text, and as expected, each have their strengths and shortfalls**

- **Other waveform codes**
  - **Non-linear FM Pulse compression**
  - **Non-linear binary phase coded sequences**
  - **Doppler tolerant pulse compression waveforms**
  - **Complementary (Golay) Codes**
  - **Welti Codes**
  - **Huffman Codes**
  - **Variants of the Barker code**
  - **Techniques for minimizing the sidelobes with phase coded waveforms**

Summary

- Simultaneous high average power and good range resolution may be achieved by using pulse compression techniques
- Modulation of long pulses, in frequency or phase, are techniques that are often for pulse compression
  – Phase-encoding a long pulse can be used to divide it into binary encoded sub-pulses
  – Linear frequency modulation of a long pulse can also be used to achieve the same effect
- Other methods of pulse coding
  – Linear recursive sequence codes
  – Quadraphase codes
  – Polyphase codes
  – Costas codes
  – Non-linear FM

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5. Nathanson, F. E., *Radar Design Principles*, New York, McGraw-Hill, 1st Ed., 1969
6. Richards, M., *Fundamentals of Radar Signal Processing,* McGraw-Hill, New York, 2005
7. Sullivan, R. J., *Radar Foundations for Imaging and Advanced Concepts*, Scitech, Raleigh, 2000

Acknowledgements

- Dr. Randy Avent

Homework Problems

- **From Skolnik, Reference 1**
  - **Problems 5-11 , 5-2, 5-3**
  - **Problems 6-17, 6-19 , 6-20, 6-21, 6-22, 6-25, 6-26, 6-27, 6-28**